Learning Control and Its Applications to Rehabilitation Robotics

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Outline

- Background: What is learning control?
- Analysis tools
- Applications to rehabilitation robotics
- Challenges
- Summary

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What is LC?

- K.S Fu in 1970 [1] pointed out, "**the gradual improvement** of performance due to **the improvement of the estimated unknown information**, this class of control systems may be called learning control systems"

[1] K.S. Fu,, IEEE Transactions on Automatic Control, April, **1970**

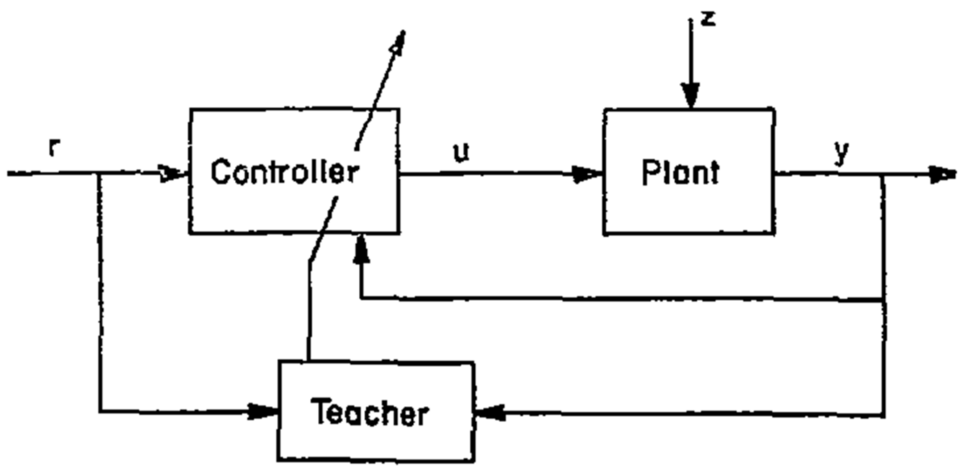

Fig. 1. Learning control system.

What is LC?

- In industry, many processes are repetitive
  - ✓ Robotic systems;
  - ✓ Batch processes;
  - ✓ High precision CNC machining;
  - ✓ Hard disc drive;
  - ✓ Milling and laser cutting
  - ✓ Traffic flow control;
  - ✓ Rehabilitation

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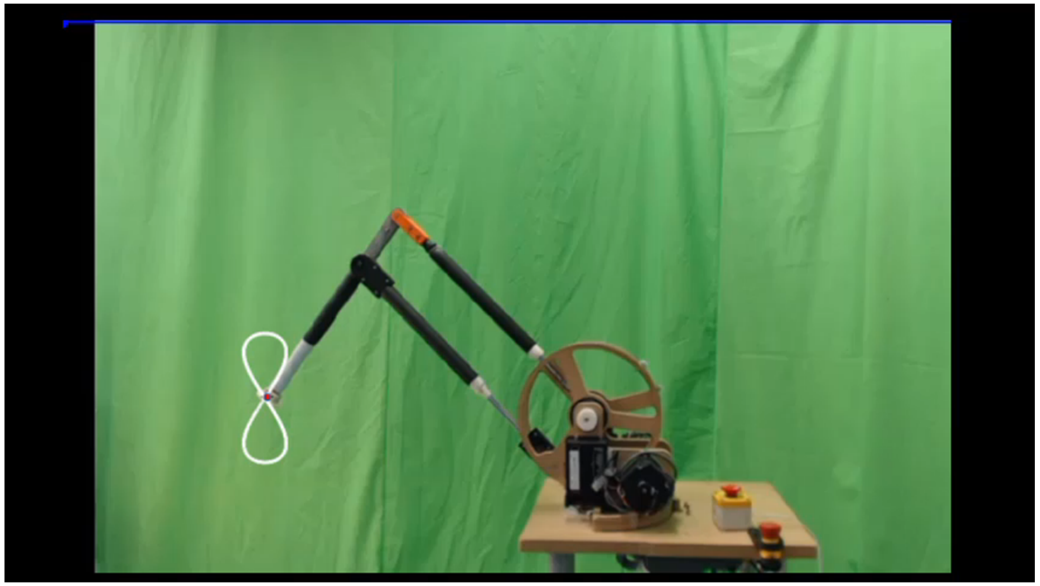
What is LC?

- Learning control (LC) is a method of tracking control for systems that work in a repetitive mode
- It was motivated by multi-pass control strategy, by J. B. Edward in 1974 [2]
- In 1984, S. Arimoto et.al [3] explicitly proposed the idea of improving performance over iterations

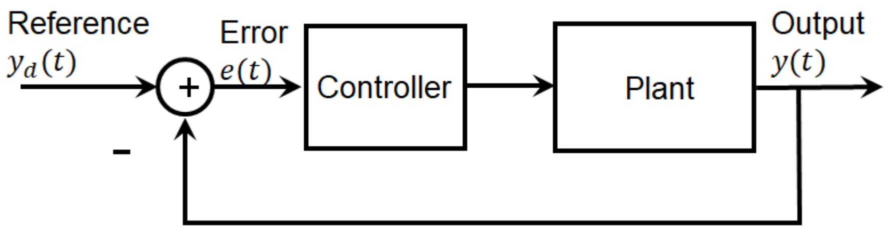


[2] J. B. Edwards, *Proc. Inst. Elect. Eng.*, 1974.
[3] S. Arimoto, S. Kawamura, and F. Miyazaki, J. Robot. Syst, 1984.

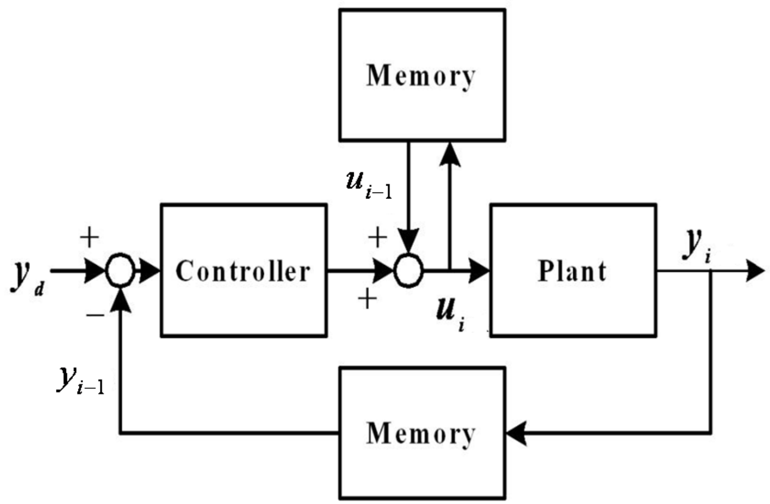
What is LC?

- The standard LC algorithm is more like a feed-forward controller (see K. Moore [4])

Feedback



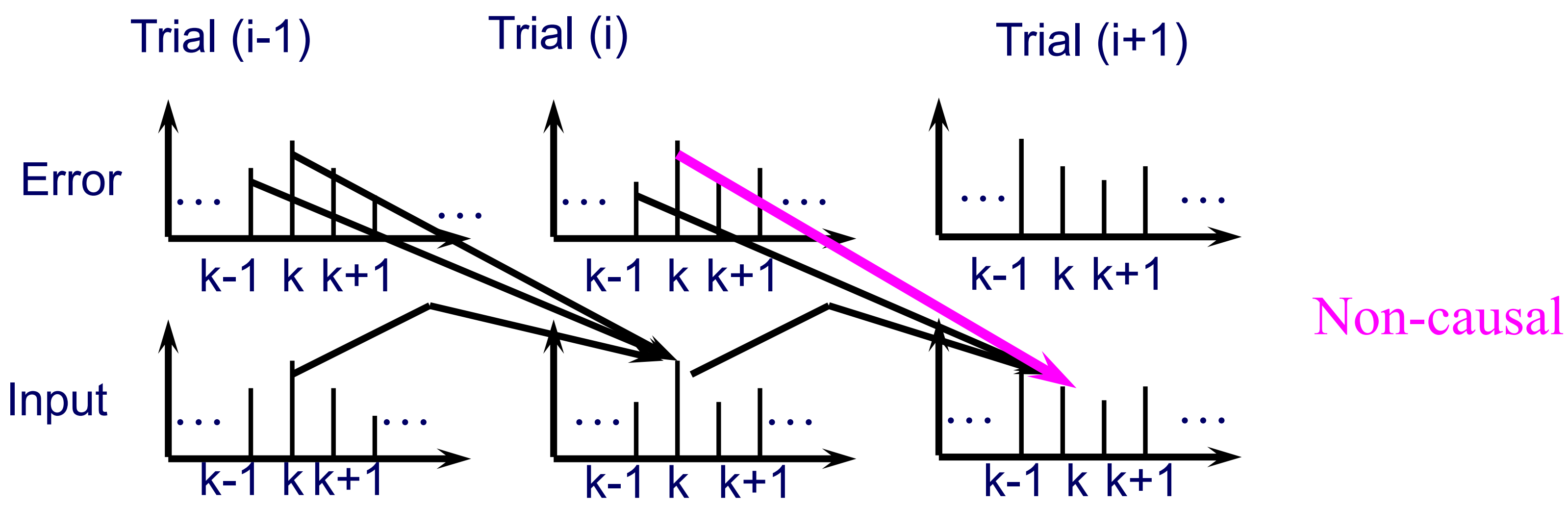
LC algorithm



[4] K. L. Moore, In Advances Industrial Control, New York: Springer-Verlag, 1993.

What is LC?

- Possible learning mechanisms



$$u_{i+1}[k] = u_i[k] + g(e_i[k+1], e_i[k], \cdots), n_s \leq N$$
$$e_i[k] = y_d[k] - y_i[k] \quad k \in \{0,1,\cdots,N\}$$

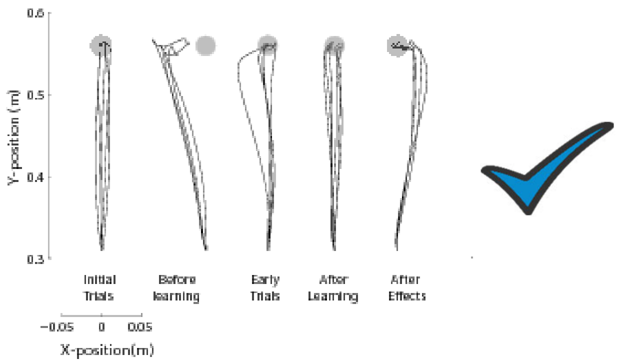
The simplest one: $u_{i+1}[k] = u_i[k] + \Gamma e_i[k]$

What is LC?

- Two types of learning control strategies

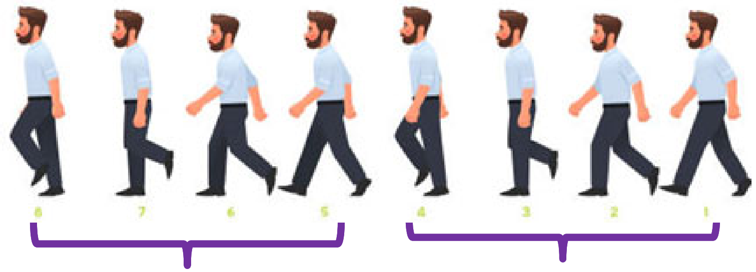
Learning Control

Iterative Learning Control $y_i[0] = y_j[0], i, j \in N_{\geq 0}, i \neq j$



See our work [5]

Repetitive Learning Control $y_{i+1}[0] = y_i[T], i \in N_{\geq 0}$



See our work [6]

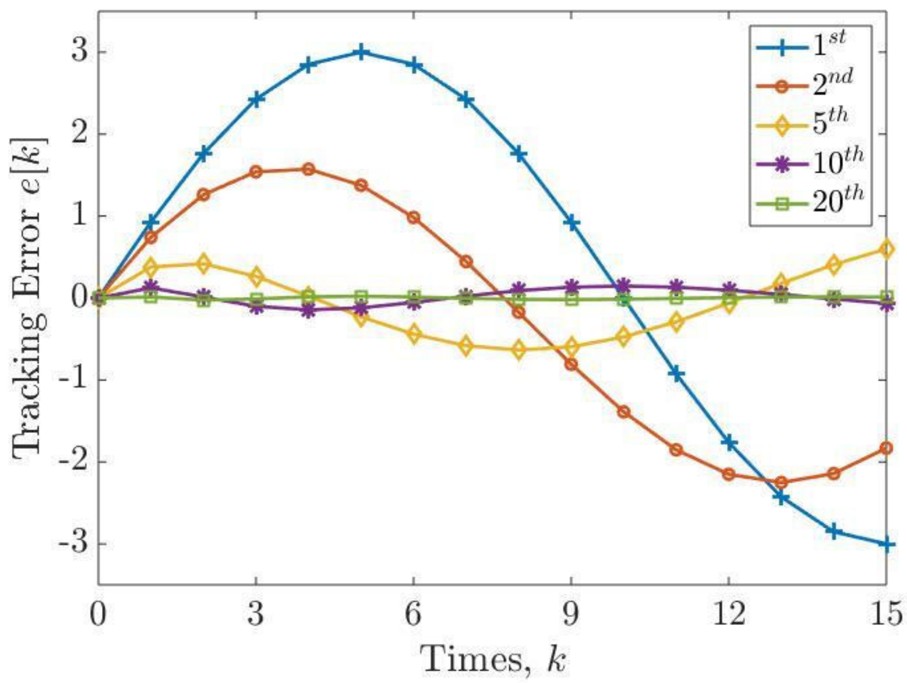
Professor Makoto's talk: vibrations in industry

[5] Zhou, Shou-Han, et. al, 2015 *IEEE Transactions on Control Systems Technology*, 2016

[6] T. Bacek et al.,International Conference on Rehabilitation Robotics (ICORR), 2022

What is LC?

- Features of ILC include
  - ✓ The same reference trajectory
  - ✓ The same time duration $k \in \{0,1,\cdots,N\}$
  - ✓ Control objective is to design a sequence of control input $\{u_i[k]\}_{i\in N_{\geq 0}}$ such that



$$\lim_{i\to\infty}|e_i[k]| = 0, \forall k = 0,1,\cdots,N$$

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Outline

- Background: What is learning control?
- **Analysis tools**
- Applications to rehabilitation robotics
- Challenges
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Analysis Tools

- Many tools have been developed over almost 50 years

Contraction Mapping (CM) ✓	Linear systems or nonlinear systems satisfying global Lipschitz continuity (GLC) condition
Composite Energy Function (CEF)	A more general class of nonlinear systems
Norm Optimal ILC	Discrete-time dynamics, linking closely to optimization
Frequency Tools	Linear systems (continuous-time and discrete-time)
2D-Theory	Widely used in discrete-time dynamics: a 2D dynamic system (time and iteration)

Analysis Tools: CM

- CM method (continuous-time): see [7]

$$\Sigma_i^{L,C}: \begin{cases} \dot{x} = Ax_i + Bu_i \\ y_i = Cx_i + Du_i \end{cases} , x_i(0) = x_0, t \in [0, T]$$

$x \in R^n$ is the state, $u \in R^m$ is the input, and $y \in R^m$ is the output (a square system)

$\forall y_d(t), t \in [0, T]$, the control objective of ILC is to design a sequence of control input $\{u_i(t)\}_{i \in N_{\geq 0}}$ such that

$$\lim_{i \to \infty} |e_i(t)| = 0, \forall t \in [0, T] \qquad e_i(t) = y_d(t) - y_i(t)$$

[7]. X. Xu and Y. Tan, Automatica, 2002.

Analysis Tools: CM

- A simple P-type ILC has the following form:

$$u_{i+1}(t) = u_i(t) + \Gamma e_i(t)$$

- Convergence analysis

$$e_{i+1}(t) = y_d(t) - y_i(t) - (y_{i+1}(t) - y_i(t))$$

$$u_{i+1}(t) = u_i(t) + \Gamma e_i(t) \qquad y_i = Cx_i + Du_i$$

$$e_{i+1}(t) = (I_{m\times m} - D\Gamma)e_i(t) - C(x_{i+1}(t) - x_i(t))$$

Contraction mapping

Perturbation

Analysis Tools: CM

- Convergence analysis (continued)

$$e_{i+1}(t) = (I_{m\times m} - D\Gamma)e_i - C(x_{i+1} - x_i) \qquad u_{i+1}(t) = u_i(t) - \Gamma e_i(t)$$

- The trick is the so-called λ-norm. For all $\lambda > 0$, it is defined as

$$\|e\|_\lambda = \max_{t\in[0,T]} e^{-\lambda t}\,|e(t)|, e(t) \in C[0,T]$$

Linear dynamics or nonlinear dynamics satisfying GLC condition (Gronwall Lemma)

$$\|x_{i+1} - x_i\|_\lambda = O\left(\frac{1}{\lambda}\right)\|u_{i+1} - u_i\|_\lambda$$

Analysis Tools: CM

- Using the updating law: $u_{i+1}(t) = u_i(t) - \Gamma e_i(t)$

$$\|x_{i+1} - x_i\|_\lambda = O\left(\frac{1}{\lambda}\right) \|u_{i+1} - u_i\|_\lambda$$

$$\|x_{i+1} - x_i\|_\lambda = O\left(\frac{1}{\lambda}\right) \|e_i\|_\lambda$$

$$e_{i+1}(t) = (I_{m\times m} - D\Gamma)e_i(t) - C(x_{i+1}(t) - x_i(t))$$

$$\|e_{i+1}\|_\lambda \le |I_{m\times m} - D\Gamma| \cdot \|e_i\|_\lambda + O\left(\frac{1}{\lambda}\right) \|e_i\|_\lambda$$

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Analysis Tools: CM

- Convergence condition: $|I_{m\times m} - D\Gamma| \le \rho < 1$

$$\|e_{i+1}\|_\lambda \le |I_{m\times m} - D\Gamma|\|e_{i+1}\|_\lambda + O\left(\frac{1}{\lambda}\right)\|e_i\|_\lambda$$

$$|I_{m\times m} - D\Gamma| \le \rho < 1 \qquad \exists\lambda \gg 1, \rho + O\left(\frac{1}{\lambda}\right) < 1$$

$$\lim_{i\to\infty}\|e_i\|_\lambda = 0$$

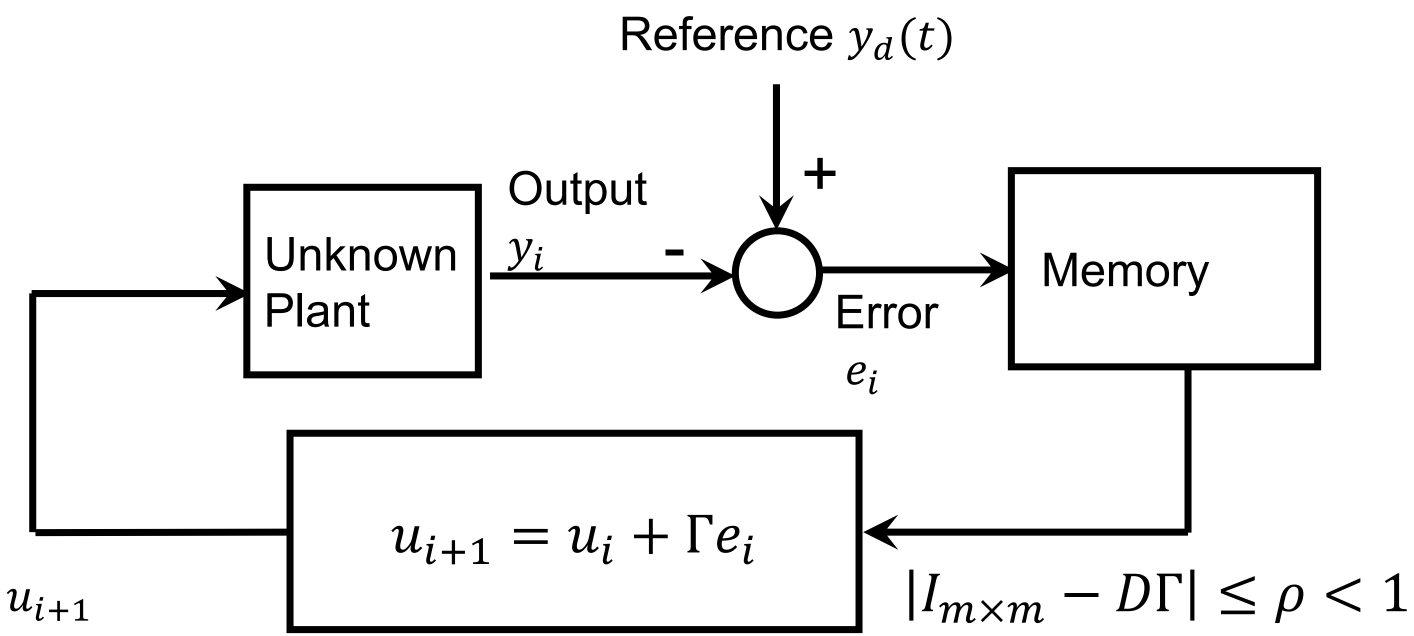
$$\lim_{i\to\infty}|e_i(t)| = 0, \forall t \in [0, T]$$

- The similar idea can be extended to linear systems (see [8]) and nonlinear systems with a higher relative degree

[8] J.-X. Xu, Y. Tan, Linear and Nonlinear Iterative Learning Control, 2003

Analysis Tools: CM

- The mapping between input and output plays a key role in convergence analysis of learning $y_i = Cx_i + Du_i$



Analysis Tools: Other Tools

- CEF is developed to analyse the convergence of ILC for nonlinear dynamics without GLC [9]. It is closely linked to stability analysis tools such as Lyapunov functions
- In [10], we showed that CM is equivalent to the CEF technique

[9] J. X. Xu and Y. Tan, IEEE Trans. Autom. Control, 2002.

[10] G. Sebastian, Y. Tan and D. Oetomo, 2019 12th Asian Control Conference (ASCC),

Analysis Tools: Other Tools

- D. H. Owens developed tools for norm-optimal ILC [11], referring to optimization techniques
- T. Oomen developed frequency analysis tools [12]
- E. Rogers and their team developed 2D theory ([13])

[11] Owens, D.H.. Springer, London, 2016

[12] Boeren, F., Bareja, A., Kok, T., & Oomen, T, IEEE/ASME Transactions on Mechatronics,2016

13] E. Rogers, K. Galkowski, A. Gramacki, J. Gramacki, and D. H. Owens, IEEE Trans. Circuits Syst. I, Fundamental Theory Appl., 2002.

Analysis Tools: Other Tools

- D. Shen developed ILC algorithms for stochastic dynamics [14]

- Z. Hou developed ILC schemes using data-driven techniques [15]

[14] D. Shen, H.F. Chen, Automatica, 2012

[15] Chi, R., Hou, Z., Huang, B., & Jin, S. Computers & Chemical Engineering, 2017

Analysis Tools: Survey Papers

- H. Ahn, et. al, [16], D. A. Bristow, et. al [17], Y. Wang, et. al. [18] published good survey papers in the area of ILC

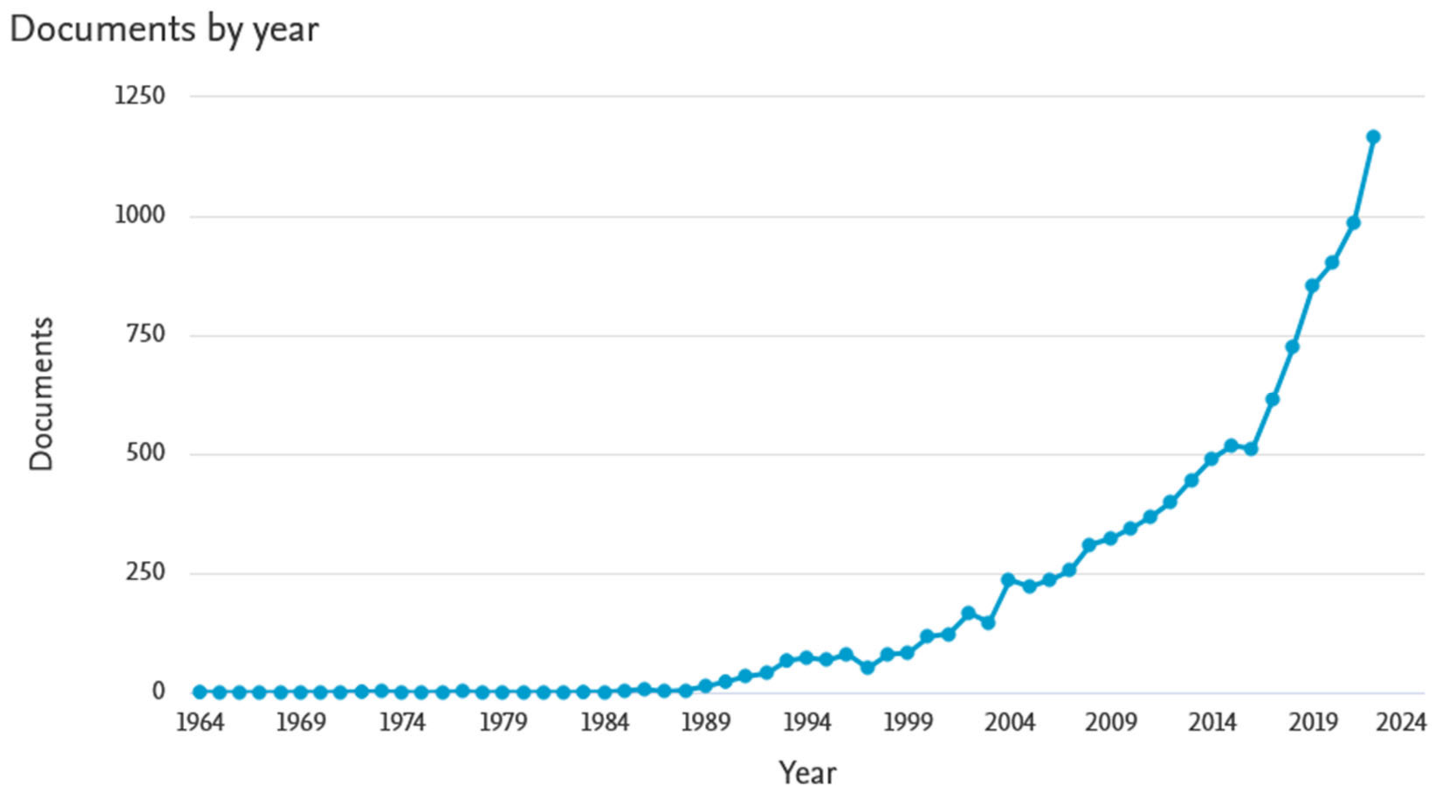
[16] Ahn, H. S., Chen, Y., & Moore, K. L., IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews), 2007.

[17] Bristow, D. A., Tharayil, M., & Alleyne, A. G. (2006 IEEE Control Systems Magazine, 2006

[18] Wang, Y., Gao, F., & Doyle III, F. Journal of Process Control, 2008

Analysis Tools

- ILC has been applied to various applications (+800 papers 2019, +1200 papers in 2022 (Scopus))



Outline

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- **Applications to rehabilitation robotics**
- Challenges
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Applications

- Rehabilitation of sensory and cognitive function typically involves methods for retraining neural pathways or **training** new neural pathways to regain or improve neuro-cognitive functioning that has been diminished by disease or trauma
- We have focused on the post-stroke rehabilitation

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Applications

- Stroke is the leading cause of mortality & disability world widely [19]
- 2 million new cases each year in China, 25% are under the age of 65 [20]

[19] Johnson, C.O., et.al., . “The Lancet Neurology, 18(5), 2019

[20] Wu, S, et.al, The Lancet Neurology, 2019

Applications

- Annual economic burden of stroke
  - ✓ Australia = $5 billion AUD [21]
  - ✓ EU = $60 billion €[22]
  - ✓ USA = $34 billion USD[23]
  - ✓ China = $40 billion RMB [24]

[21] Deloitte Access Economics, National Stroke Foundation", 2013

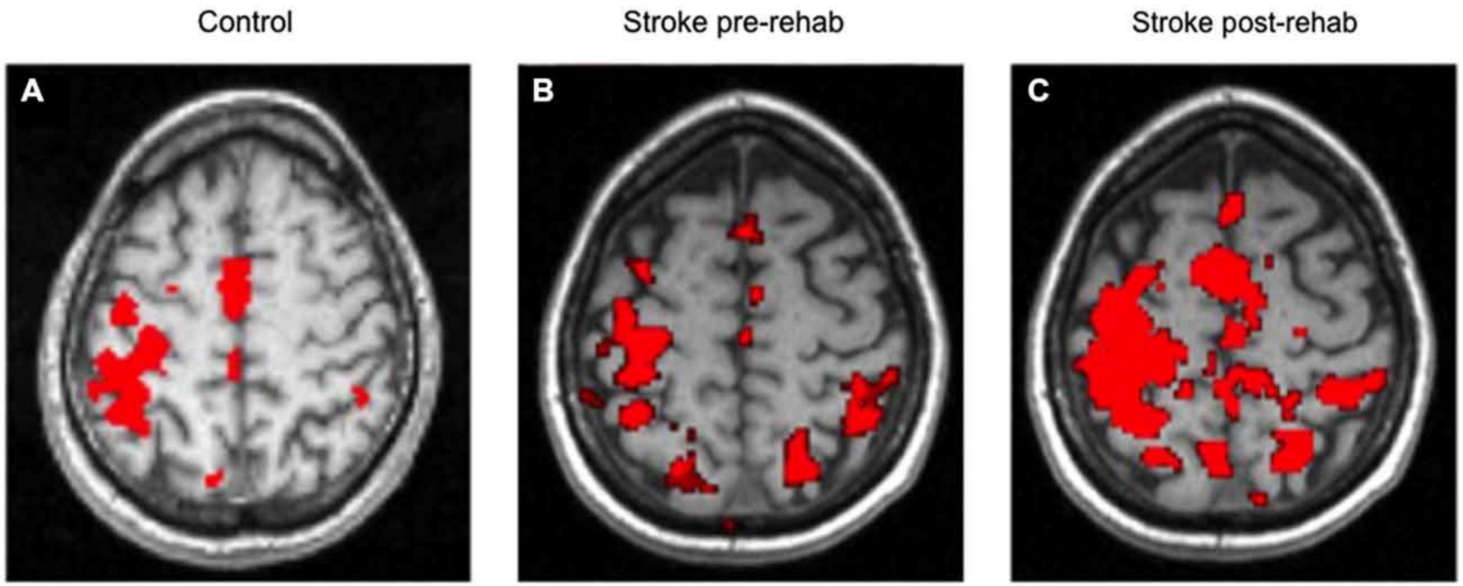
[22] Luengo-Fernandez, R. et al. European Stroke Journal, 2019

[23] Benjamin EJ, Blaha MJ, Chiuve SE, et al. Circulation. 2017

[24] Liu, L., et al., Stroke, 2011

Applications

- Brain has the ability to change and adapt [25]



fMRI changes following 12-week UL rehabilitation program (chronic stroke)

[25] Purves D, Augustine GJ, Fitzpatrick D, et al., 2001

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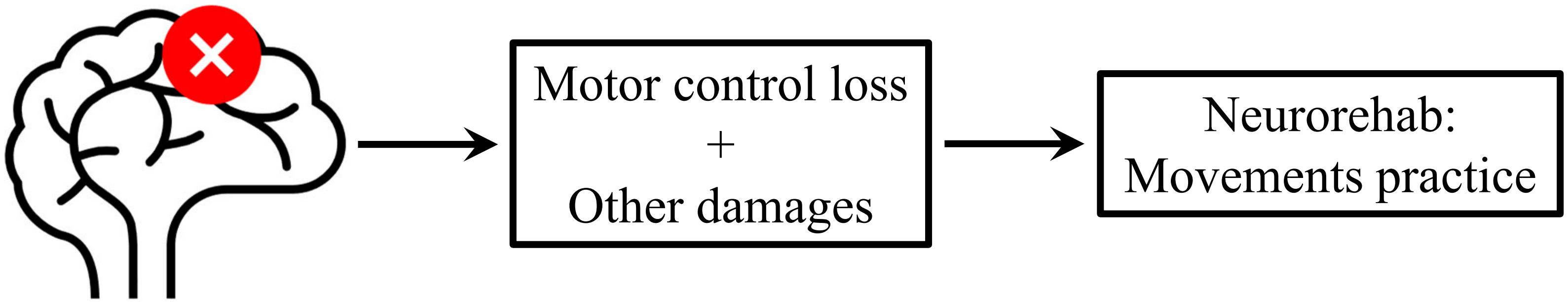
Applications

- Physiotherapy and Occupational Therapy as main treatment



- By repeating the simple tasks many times, it is possible to "recover"

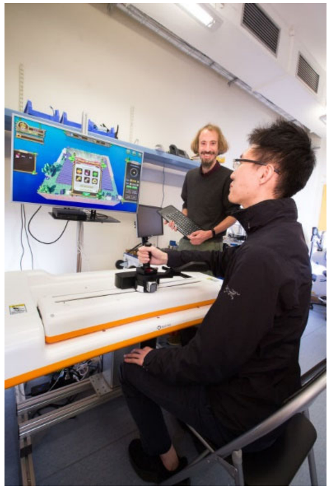
- ILC: repeating the same task will improve the performance in the presence of unknown human model and huge human variations (data-drive, personalized treatments are needed)

Applications

- ILC in designing control: Functional Electrical Stimulation (FES) + robotics

Applications

- We have worked on robotics (upper limb)



EMU

In 2022, EMU received the Red Dot Design Award, German iF Design Award, the Japan 2022 Good Design Award.

In 2023, it received IEEE-IFR Innovation and Entrepreneurship in Robotics and Automation Award (IEEE Robotics and Automation Society)

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Applications

- Lower limb rehabilitation robotics

Applications

- Robotics can provide high density (30 minutes are required, but in Australia, only 9 minutes are delivered) + measurements

Recommended dose is not sustainable by conventional practice

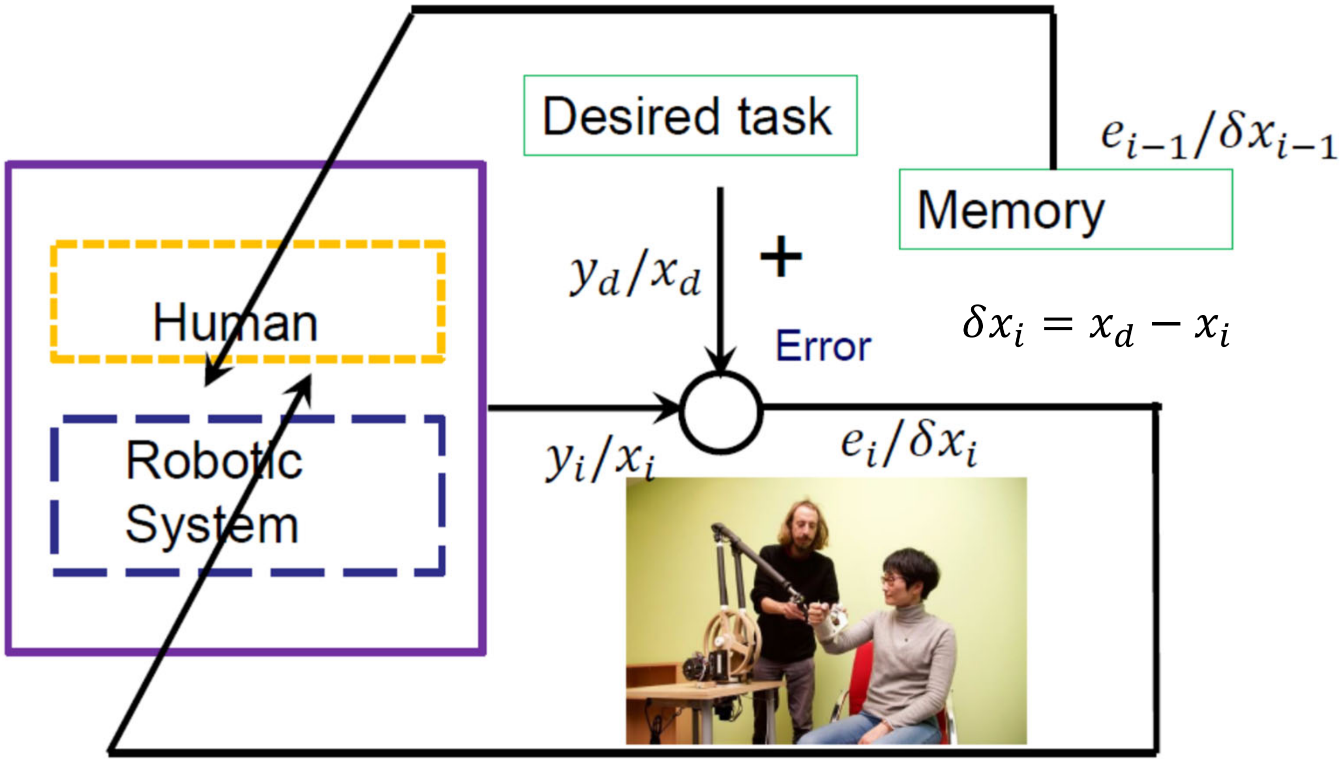
- Conventional:
  1.21 reps per min for stroke [26]
- Robot:
  17 reps per minute [28]

[27] Kimberley, T.J., Samargia, S., Moore, L.G., Shakya, J.K. and Lang, C.E., 2010.

[28] Lo, A.C., Guarino, P.D., Richards, L.G., Haselkorn, J.K., Wittenberg, G.F., Federman, D.G., Ringer, R.J., Wagner, T.H., Krebs, H.I., Volpe, B.T. and Bever Jr, C.T., *New England Journal of Medicine*, 2010

Applications

➢ Question: can more assistance provide better learning?

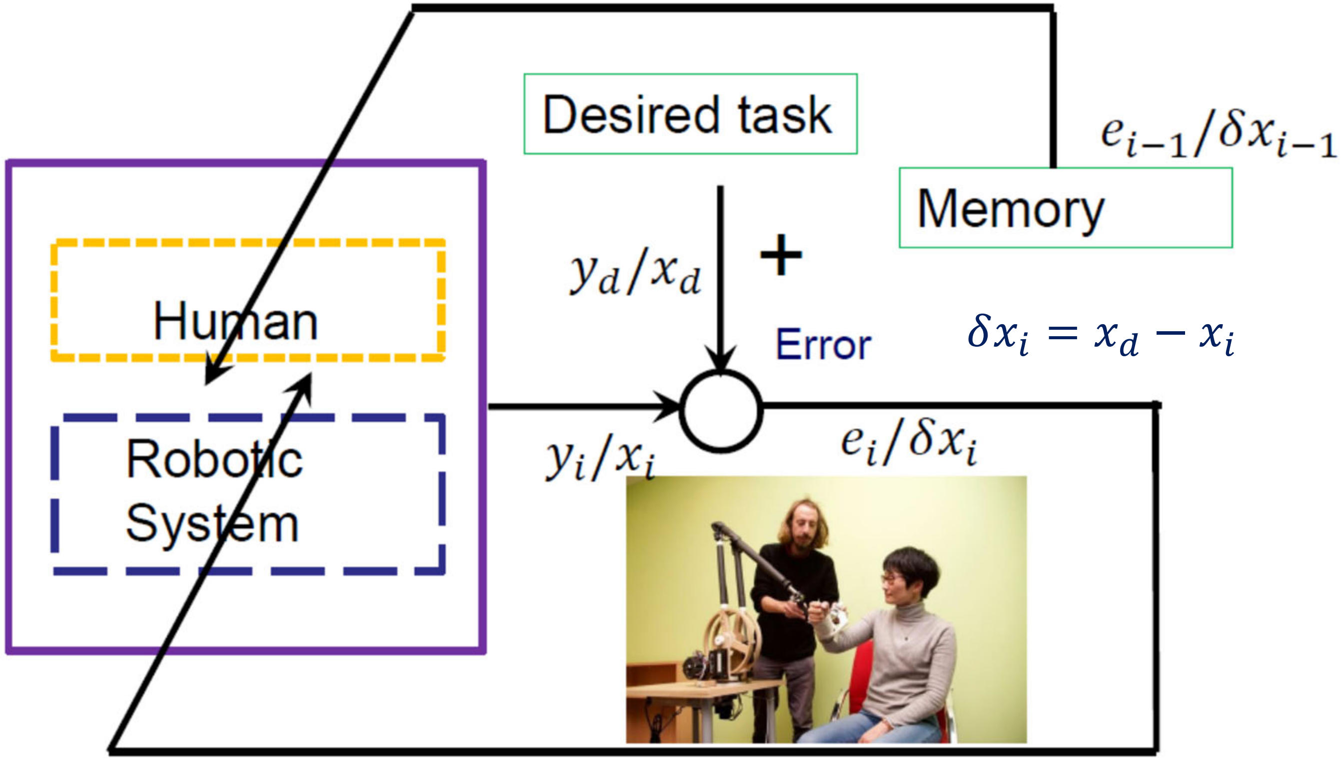


Applications

- Question: can more assistance provide better learning?

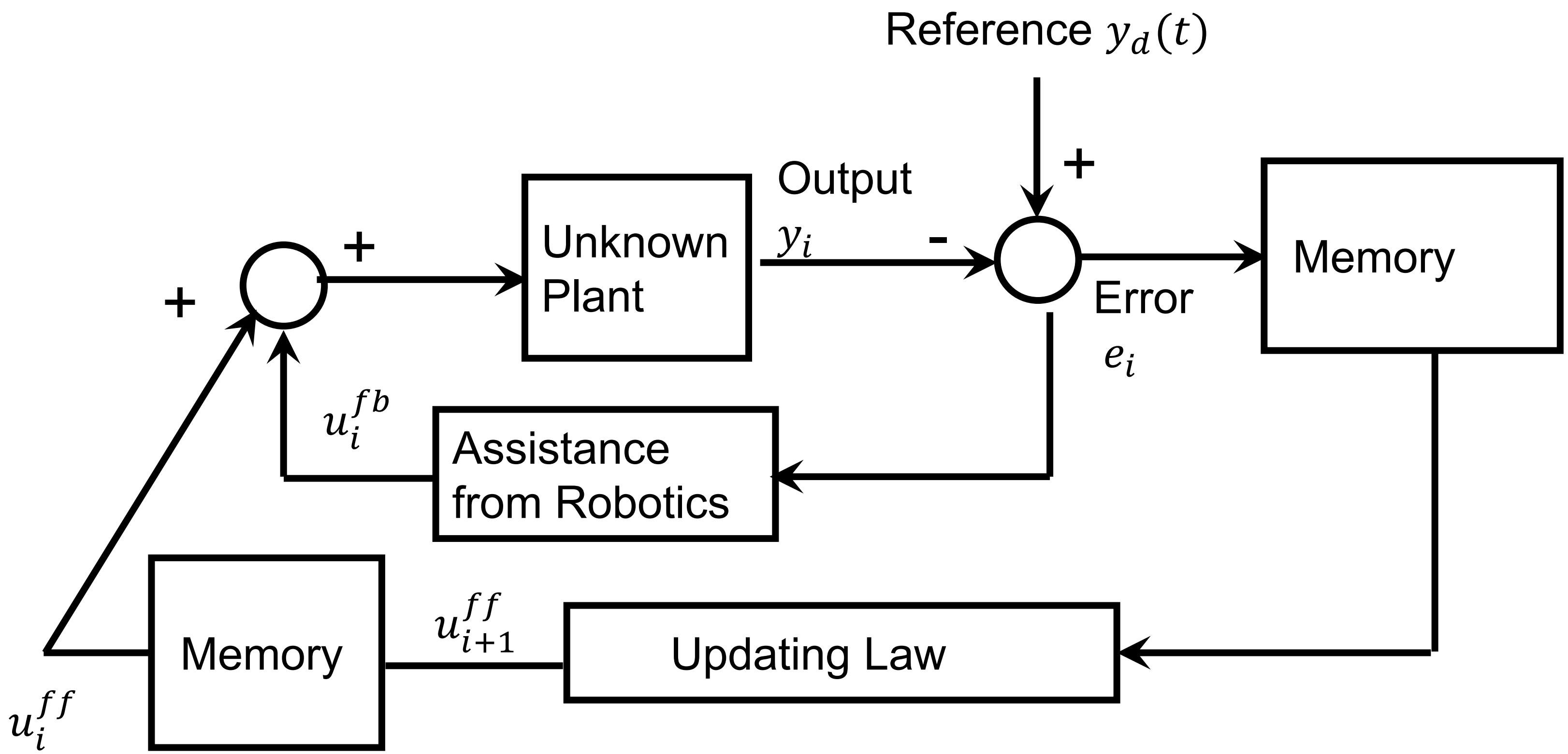
Human learns to perform a task iteratively

Robotic system provides feedback



Applications

- It is a feedback (the robot) +feed-forward (the human): similar to Professor Makoto's talk (one loop is not controllable)



Applications

- Question: can more assistance provide better learning?
  - ✓ We want assist-as-needed
  - ✓ A typical engineering solution is to gradually reduce the assistance from the robot [28]
  - ✓ What we want is to have an optimal assistance from the robot

[28] Meng W, Liu Q, Zhou Z, et al.. Mechatronics 2015

Applications

- We show it mathematically
  - ✓ It is assumed that the overall system is an LTI (this assumption holds locally)

$$\Sigma_i^{Overall}: \begin{cases} \dot{x} = Ax_i + B\left(u_i^H + u_i^R\right) \\ y_i = Cx_i = x_i \end{cases} \qquad x_i(0) = x_0, t \in [0, T]$$

  - ✓ It is assumed that there is the desired reference trajectory satisfying the following dynamics (matching condition)

$$\Sigma_d: \dot{x}_d = Ax_d + Bu_d$$

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Applications

- For a given x_d, the tracking error is $e_i(t) = x_d(t) - x_i(t)$

- If $CB \neq 0_{n\times n}$, the system has the relative degree 1

- Assuming human learning uses D-type ILC:

$$u_{i+1}^H(t) = u_i^H(t) + \Gamma\,\dot{e}_i(t) \text{ (see our work [29])}$$

$$|I_{m\times m} - CB\Gamma| \leq \rho < 1 \quad \text{Convergence condition}$$

[29] S.-H. Zhou, D. Oetomo, Y. Tan, E. Burdet, and I. Mareels, IEEE Transactions on Biomedical Engineering, 2012.

Applications

- It is assumed that the robot utilizes a state feedback controller to aid

$$u_i^R(t) = Ke_i(t)$$

- It has the following error dynamics

$$\dot{e}_i = (A - BK)e_i + Bu_i^{ff} \qquad |I_{m\times m} - CB\Gamma| \le \rho < 1$$

- Simulation results showed that a higher feedback gain will lead a lower convergence speed, though the tracking error of the first iteration is smaller

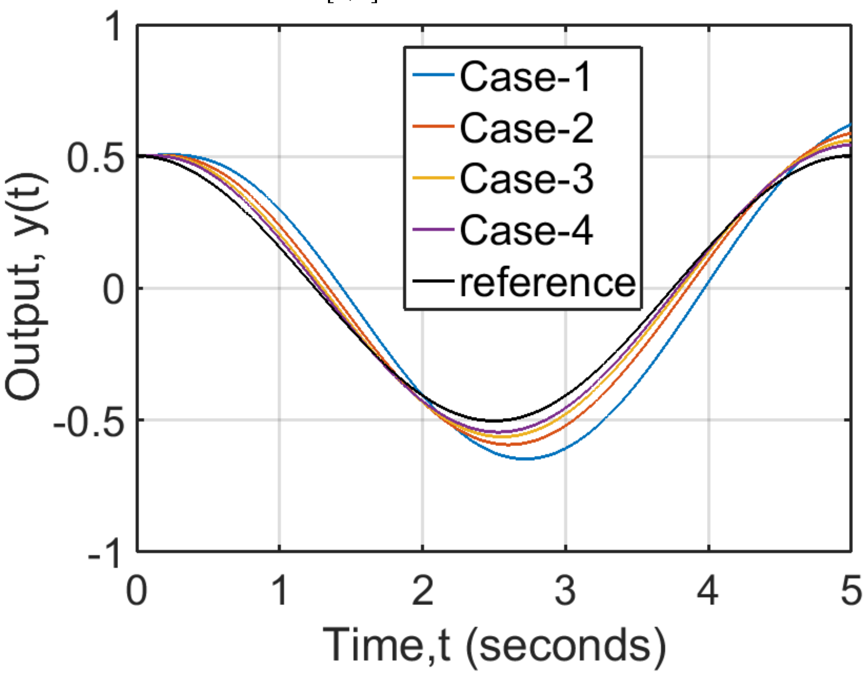
Applications

➢ Simulation results

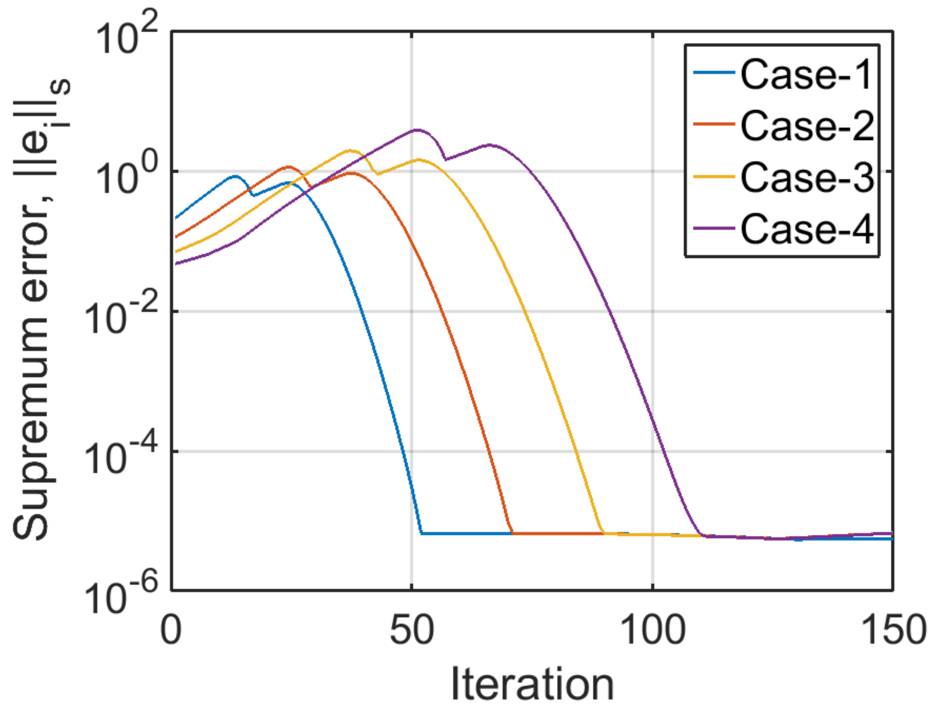
	Case-1	Case-2	Case-3	Case-4
CL- poles	$-2 \pm j$	$-3 \pm j$	$-4 \pm j$	$-5 \pm j$
K	[6 4]	[11 6]	[18 8]	[27 10]

We observed that having complex conjugate poles converges faster (PE condition)

$$\|e_i\|_s = \max_{t \in [0,T]} |e_i(t)|$$



Tracking performance at the 1st iteration



Tracking performance over iterations

Applications

- Mathematical analysis show how dynamics from the feedback will affect the convergence

$$\left\|\dot{e}_{i+1}\right\|_s \leq \rho\left\|\dot{e}_i\right\|_s + |C||B|\phi(A-BK)\left\|\dot{e}_i\right\|_s \qquad \left\|e\right\|_s = \max_{t\in[0\ \ T]} |e(t)|$$

where $\phi(A-BK) := \frac{|A-BK|}{|\lambda_R(A-BK)|}|T_\mu||T_\mu^{-1}|$ is the dynamic influence

For a given positive constant μ, $T_\mu(A-BK)T_\mu^{-1}$ is the modified Jordan form

$$\lambda_R(A) = \max_{i=1,..n}\{Re(\lambda_i)\}$$

where λ_i is the i^{th} eigenvalue

Applications

- We can minimize the dynamic impact to reach monotonic convergence

$$\min_{K \in S} \phi(A - BK)$$

where S- set containing all stabilising K

This condition will be re-considered motivated by Rodolph's talk

- It is a highly nonlinear and non-convex optimization problem

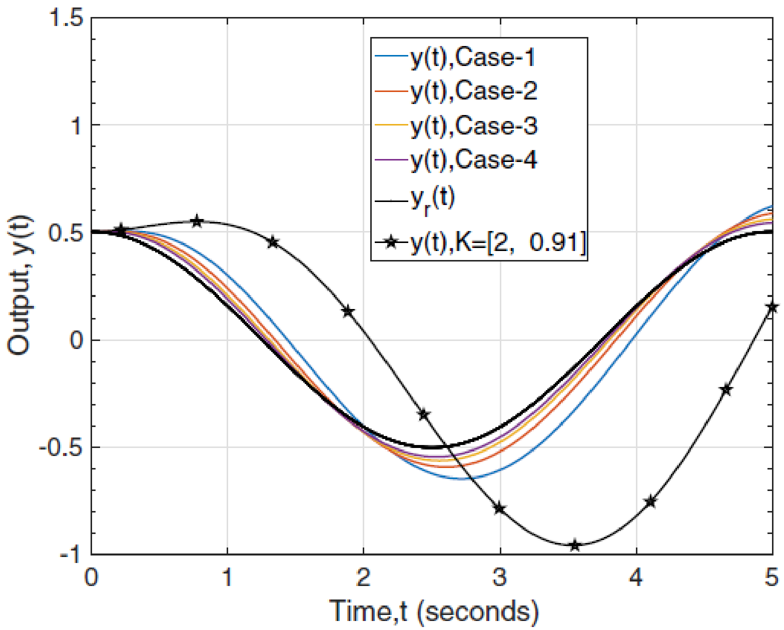
- The solutions might not be unique (see our work[30])

[30] Gijo Sebastian, Ying Tan, Denny Oetomo and Iven Mareels, ASCC, 2017

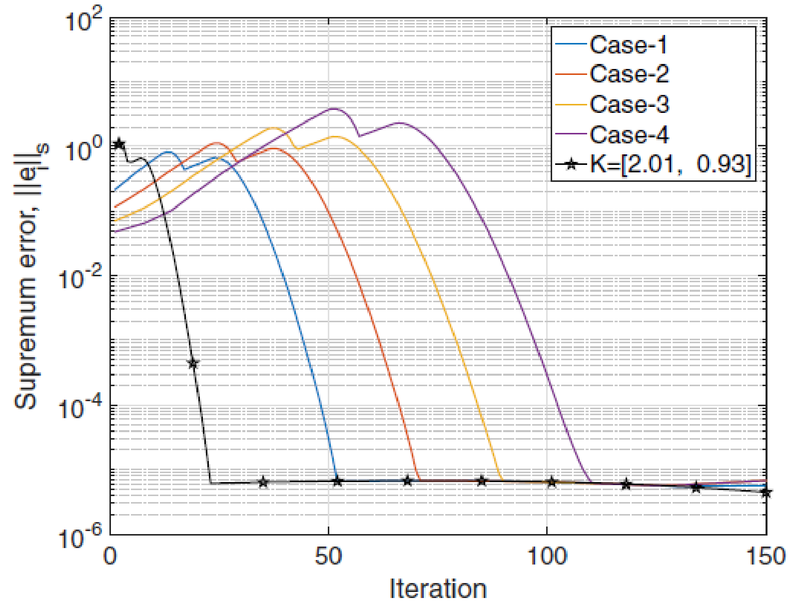
Applications

- If we know (A, B, C), we can use constrained optimisation using MATLAB inbuilt function 'fmincon' for illustration to obtain $K^* = [2.01,\ 0.93]$

Output response using feedback only



Convergence

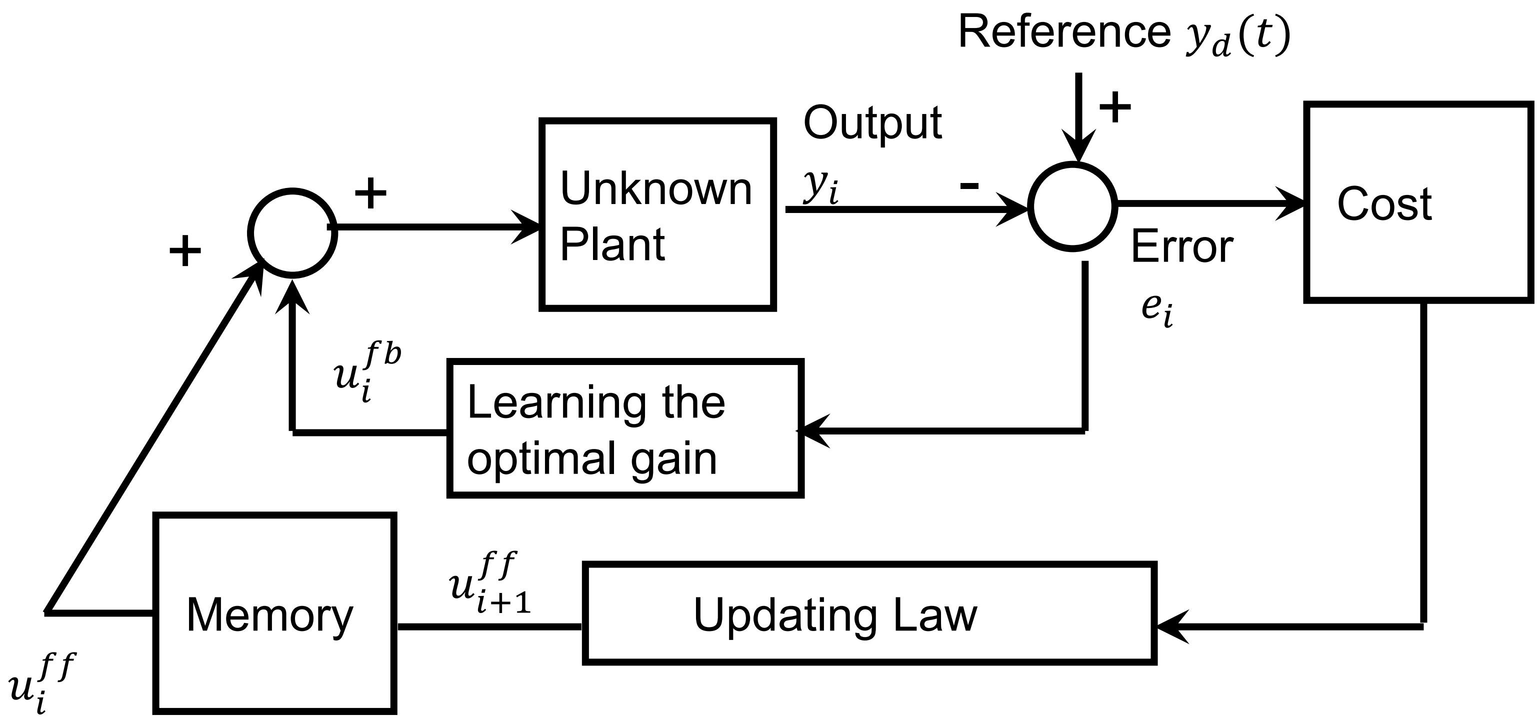


Applications

- For an unknown system, a practical algorithm is proposed:
  - ✓ Identifying the nominal personalized model from measured data
  - ✓ An off-line optimization technique is used to find the optimal feedback gain $\widehat{K}^*$ locally or globally
  - ✓ An on-line tuning algorithm is used to find K^* from initial value $\widehat{K}^*$ with the appropriate choices of the cost function (to ensure some local convergence)

Applications

- The diagram of such a solution



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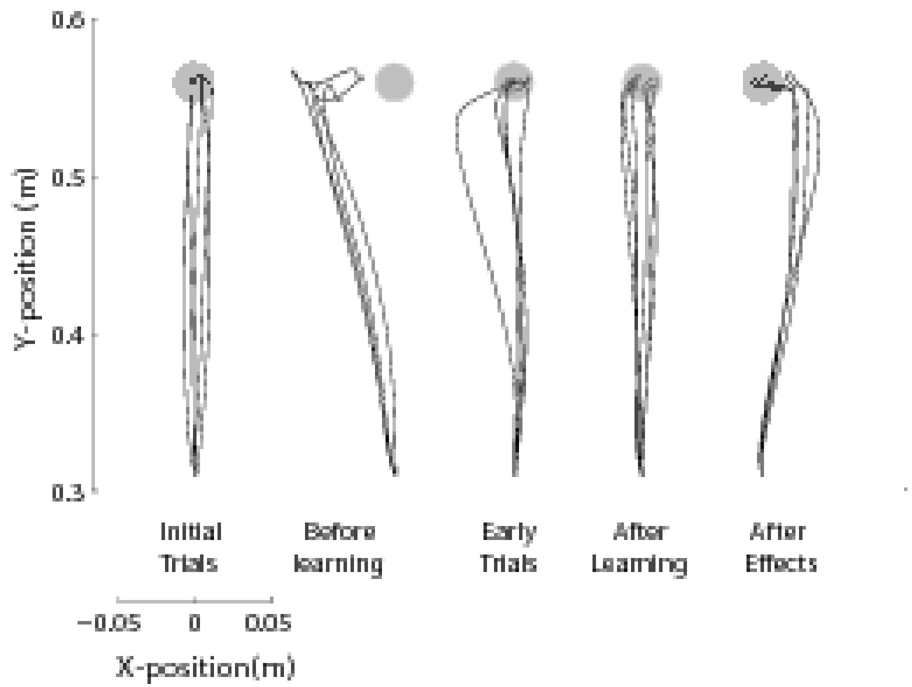
Challenges: ILC Design

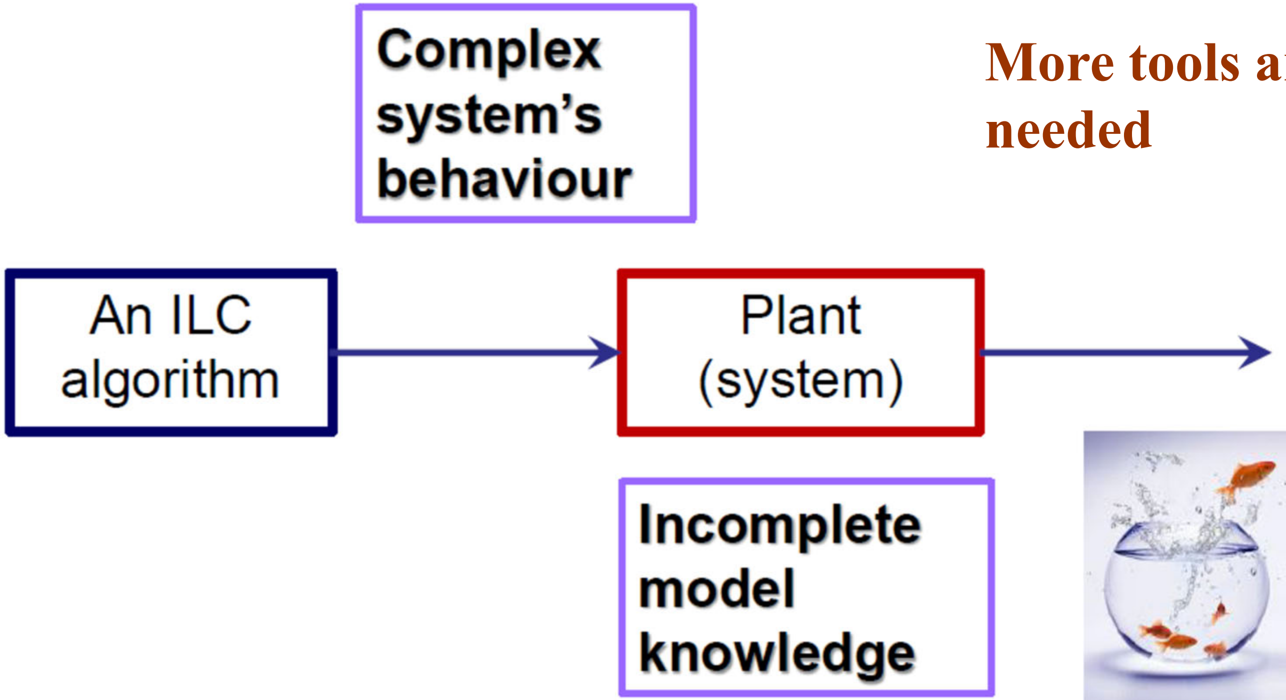
- The hybrid nature in ILC (continuous in finite time domain + discrete in iteration domain) makes analysis harder



Hybrid framework? Signal Set?

Challenges: ILC Design

- Design, convergence, and **performance** analysis are still very challenging for complex systems



More tools are needed

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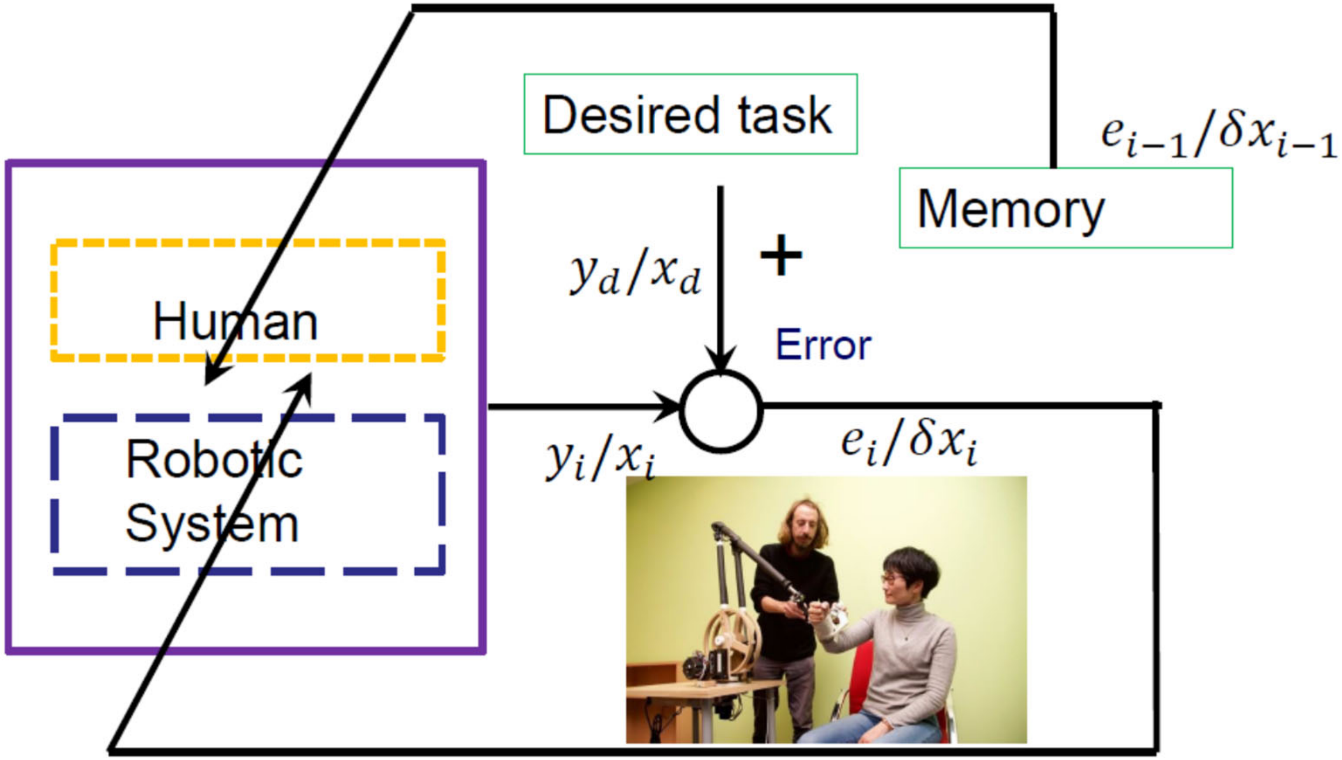
Challenges: Rehab Robotics

- The overall system (human + robot) : unknown, complex large human variations
- The control objective is not clear

Improving human learning

Robotic system provides feedback

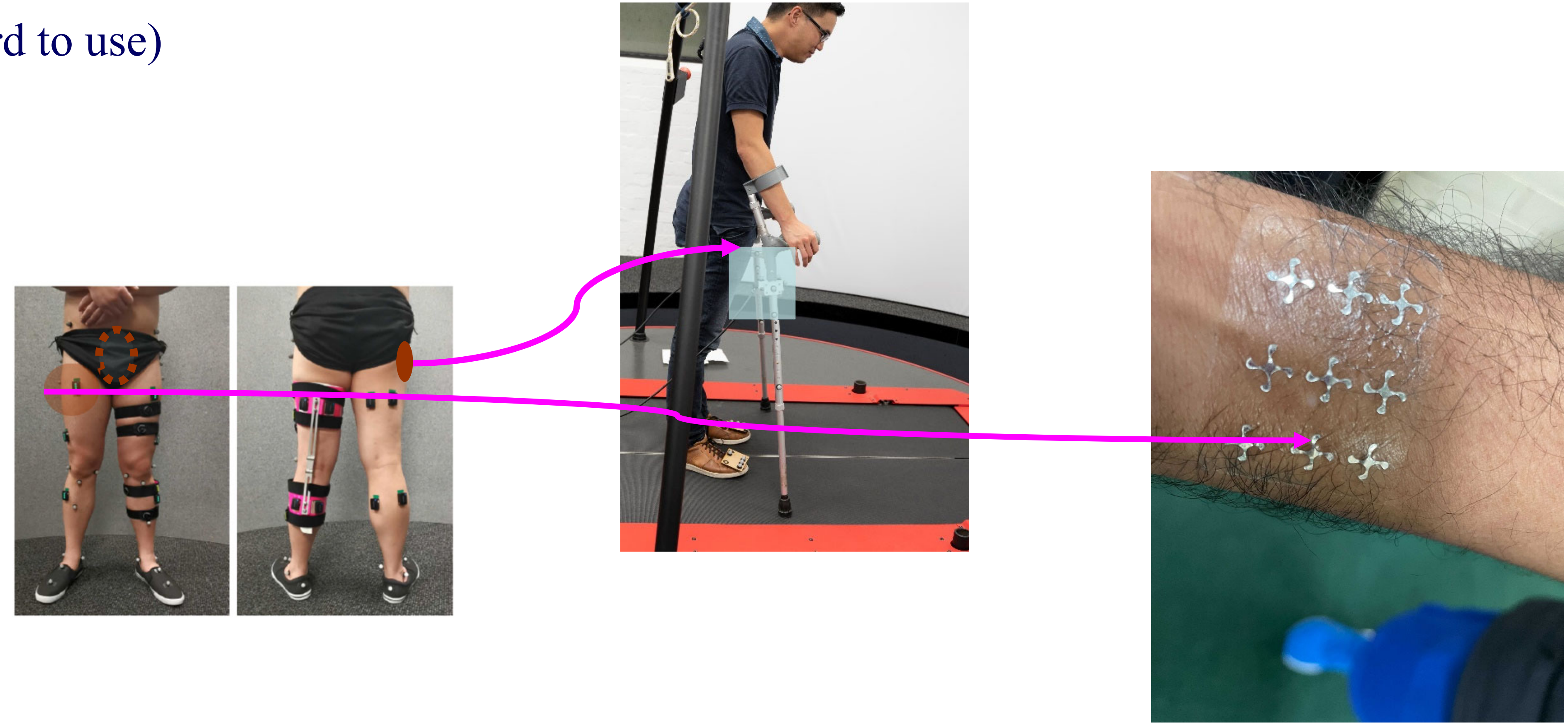


Challenges: Rehab Robotics

- Is human optimizing? If so, what is the cost function?

Challenges: Rehab Robotics

➢ Can we measure the performance in a non-laboratory setting? (CAREN system is expensive and hard to use)



Challenges: Rehab Robotics

- Trajectory learning? Task learning? (Implicit learning without knowing the tracking error [31]

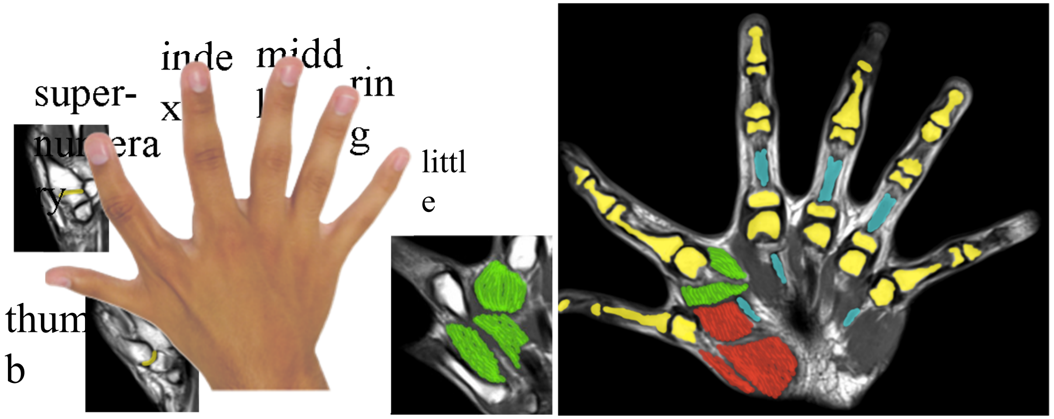
- Learning at different Co-adaptation, co-learning, and co-evolution

Similar concept was mentioned in Professor Dai's talk. Designing the long-term autonomy is needed as in Professor Magnus's talk

[31] Y. Xu, V. Crocher, J. Fong, Y. Tan and D. Oetomo, IEEE Transactions on Neural Systems and Rehabilitation Engineering, 2021

Challenges: Rehab Robotics

- What are the appropriate after learning behaviours when the robotics are not completely transparent? More natural movement? Can we use human language to control **(similar idea was mentioned in Professor Hong's talk)**



Mehring, Akselrod et al. 2019 Nature Communications

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Outline

- Background: What is learning control?
- Analysis tools
- Applications to rehabilitation robotics
- Challenges
- **Summary**

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Summary

- ILC can be applied to many engineering applications including rehabilitation robots
- How to deliver an optimal assistance from robotics along long rehab procedure is challenging (more theoretical developments are needed)
- We need to work with clinicians, patients, carers, roboticist, control experts, bio-mechanist, statisticians and engineers
- We are learning iteratively as well

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